Vortex Methods for Flow Simulation

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Essentials

- Numerical technique to solve the Navier-Stokes Equations
- Suitable for Direct Simulation and Large-Eddy Simulation
- Uses vorticity (curl of the velocity) as a variable
- Computational elements move with the fluid velocity

Advantages

- Computational elements only where vorticity is non-zero
- No grid in the flow field
- Only 2D grid on vehicle surface
- Boundary conditions in the far field automatically satisfied

Vortex Method as a Flow Model

Previous limitations (1960s and 70s)

- Inviscid model – dynamics of the boundary layer ignored
- Computationally limited – $O(N^2)$ operations per time step
- $N =$ only a few hundred to a few thousand computational elements feasible
- Dynamics of the wake and force coefficients dependent on adjustable parameters

Recent Developments (90s)

- Viscous effects treated accurately
- Fast Vortex Algorithm – $O(N)$ operations per time step
- $N =$ one to 10 million computational elements feasible
- Dense system of computational elements solves fluid equations
  - Direct simulation for low Reynolds number
  - Large-Eddy simulation for high Reynolds number
- Large-scale parallel computing

Treatment of Surface Vorticity

Standard Panel Method for N Panels

- Low order accuracy – first order accurate
- Computationally and storage limited – $O(N^2)$ matrix elements computed and stored with $O(N^2)$ operations per time step
- Only $N = 10,000$ to $20,000$ feasible

Advanced Panel Method

- High accuracy – third order accurate
- Computationally efficient – $O(N)$ storage locations with $O(N^{3/2})$ operations per time step [can go to $O(N \log N)$ or $O(N)$]
- $N = 10,000$ to $20,000$ no problem
- Triangular mesh with automatic refinement

Large-Eddy Simulation

Direct Simulation not Sufficient (1990s)

- Direct Simulation possible for Reynolds No.=10^3 to 10^4
  (at parking speeds – 0.01 mph)

- $N = 10^{14}$ elements (approx 400,000 GBytes) required for Reynolds No.=20×10^6
  (at highway speeds – 70 mph)

Large-Eddy Simulation Required

- Treatment of small-scale (subgrid-scale) turbulence in the wake

- Treatment of small-scale turbulence in the boundary layers

- Treatment of fluidic actuators, blowing/suction, vortex generators and other flow control devices

Status / Future Work

- Fast adaptive panel method
- Panel method interaction with outer flow via method similar to Verzicco
- Subgrid stress model for Large-Eddy Simulation
- Implementation of the Vortex Method for complex geometries